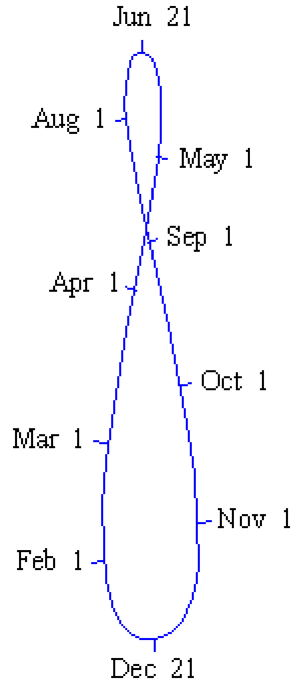
Eccentricity, Obliquity, and the Analemma's Width by John Holtz



As explained on the previous page, the Earth's elliptical orbit (eccentricity) and tilt of the axis relative to the orbit (obliquity) cause the analemma to have some width. To calculate the relative importance of these two effects, let's imagine that there are three suns in the sky:

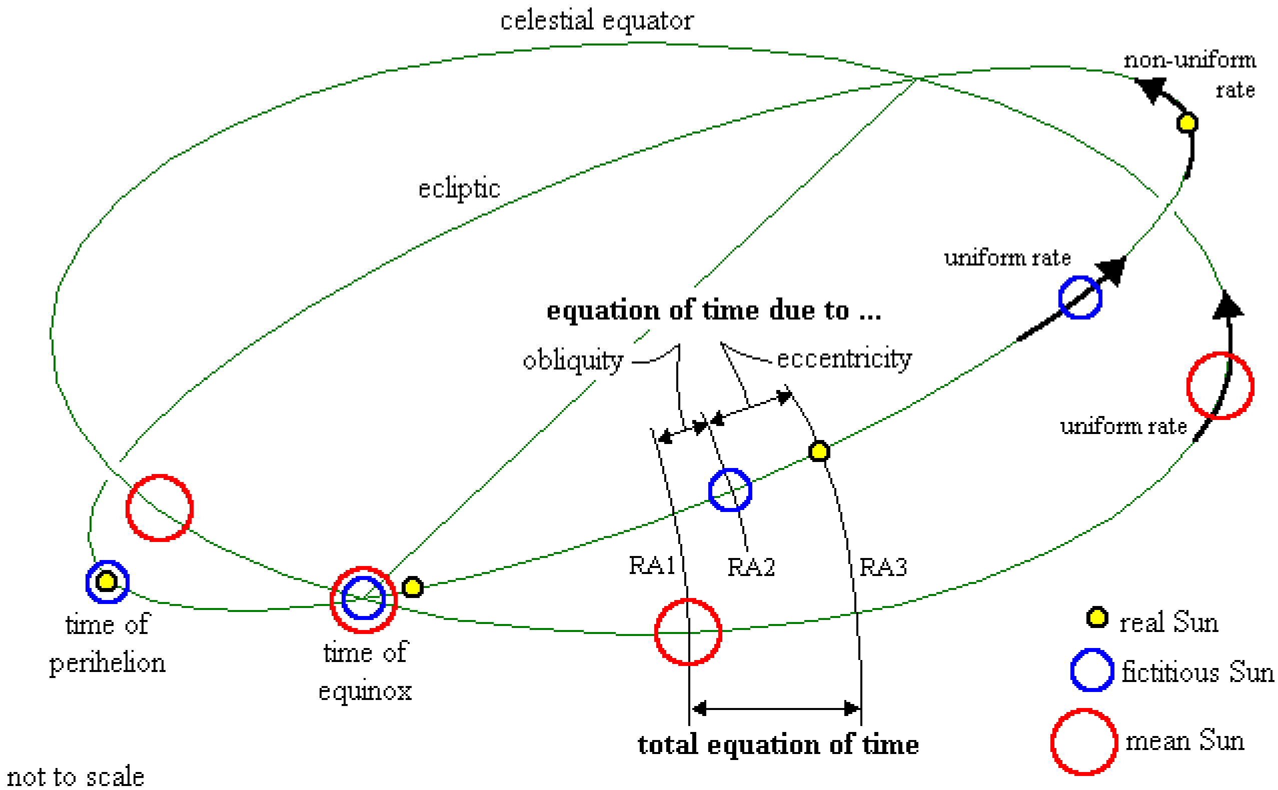
1. the real Sun. Due to the eccentricity of the Earth's orbit and obliquity of the axis, the real Sun appears to move along the ecliptic at a non-uniform rate.
2. a fictitious sun, which appears to move along the ecliptic at a uniform rate. In other words, the eccentricity of the Earth's orbit is made to be zero. Imagine the fictitious and real suns to be at the same location at the time of perihelion.
3. a mean sun, which appears to move along the equator at a uniform rate. In other words, the eccentricity and obliquity are made to be zero. Imagine the mean and fictitious suns to be at the same location when the fictitious Sun reaches the vernal equinox.

Because our clocks run at a constant rate, they need to be set to a source that moves at a uniform rate. Because objects in the sky appear to move parallel to the celestial equator due to the Earth's rotation, our source should also move parallel to the equator (or move on the equator). The real Sun in the obvious choice, except it does not meet either of these requirements. Only the mean sun meets these requirement. The horizontal difference between the real Sun and the mean sun is what gives the analemma its width.

The fictitious sun is not required since the mean sun could be defined as coinciding with the real sun at the equinox. However, including the fictitious sun enables us to separate the effects due to the obliquity and eccentricity.

The figure below shows the three different suns on their respective orbit. The distance between any two of the suns could be measure in degrees, but astronomers (and sundial makers) like to measure the difference in units of time. In other words, how long does it take the Earth to rotate so that the Right Ascension (RA) of one of the suns is replaced by the Right Ascension of another sun. When measured in units of time, this difference is known as the *equation of time*.

Defining the Equation of Time



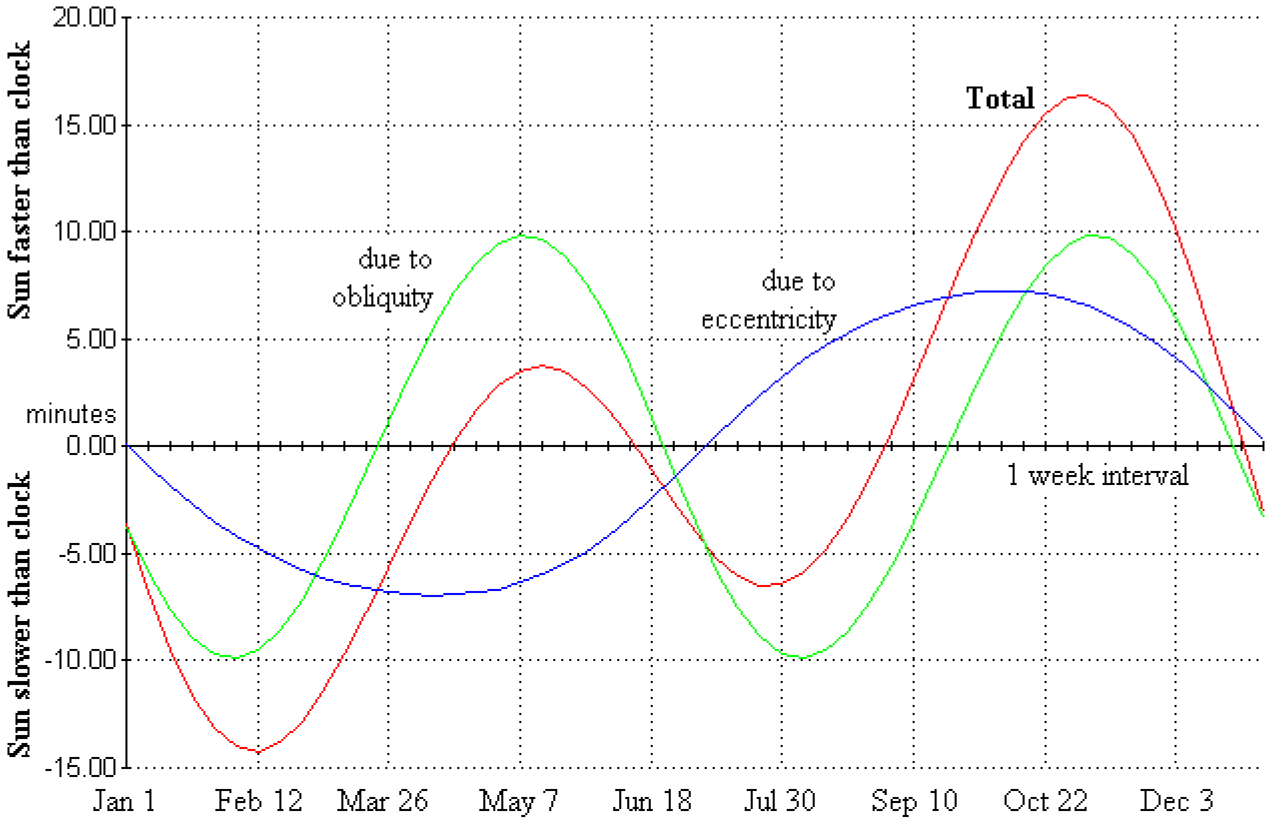
This figure shows the apparent orbit of the Sun in the sky: the definition of the ecliptic. (It is really created by the Earth's orbit around the Sun. In this figure, the Earth, not shown, would be a stationary object at the center of the orbit.) Three different suns are shown. Because the Earth is rotating, all three suns appear to move across the sky. The length of time between one sun crossing the meridian and another sun's crossing is defined as the "equation of time". The positions of the three suns are not to any scale except for the two places where they are defined to be the same.

To calculate the three values for the equation of time — that due to eccentricity, due to obliquity, and the total — I took the simplified approach. From the above diagram, it is clear that the total equation of time is equal to the sum of the equation of time due to the eccentricity and the obliquity. Since the total is well known from a variety of sources and equations (I obtained mine from The Astronomical Almanac), I only needed to calculate one of the remaining two, and simple subtraction would give the third! Calculating the equation of time between the real Sun and the fictitious sun is somewhat complex: it involves the same problem that Kepler and everyone else trying to solve elliptical orbits has encountered. On the other hand, calculating the equation of time between the fictitious and mean suns involves nothing more complicated that spherical trigonometry. The method goes something like this:

1. By definition, the fictitious and real suns are together at the time of perihelion.
2. Based on the speed of the fictitious Sun (360° per 365.25 days) and the position of perihelion (about ecliptic longitude 283°), calculate when the fictitious Sun reaches the equinox (slightly later than the real Sun).
3. By definition, the mean and fictitious suns are together when the fictitious sun is at the equinox.
4. For any other date, calculate the ecliptic longitude of the fictitious sun and the equatorial longitude (or Right Ascension) of the mean sun.
5. Convert the position of the fictitious sun from ecliptic coordinate to equatorial coordinate using the equation tan(right ascension) = cos(obliquity)*tan(ecliptic longitude). Note that the declination of the fictitious sun is not important.

6. The difference between the mean sun and the fictitious sun gives the equation of time due to obliquity. This difference is converted to minutes.
7. The equation of time due to eccentricity is calculated from the total equation of time minus the equation of time due to obliquity.
8. All three equations of time are plotted.

Graph of the Equation of Time



The green line shows the equation of time due to the effects of the Earth's obliquity (tilt), and the blue line shows the equation of time due to the effects of the Earth's eccentricity (elliptical orbit). The red line shows the total equation of time: the difference between the time shown by our clocks and the real Sun. (Another factor to add is the difference due to your longitude. That is, everyone's clock within a given time zone is set to the same time, but the Sun implies a different time at different longitudes. This difference would not change the shape of the curves; it would merely change the numbers on the vertical axis by a fixed amount.)

Interestingly, the line for the total equation of time should look very similar to the plot of the analemma. In fact, if you fold the plot in half about the 0 minute line, then fold it again about the solstices (June 21 and Dec 21), the red line will form the figure-eight analemma. Stretch the graph appropriately along the date and you will have the analemma exactly!

Page created and maintained by John Holtz. Comments are welcomed.

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